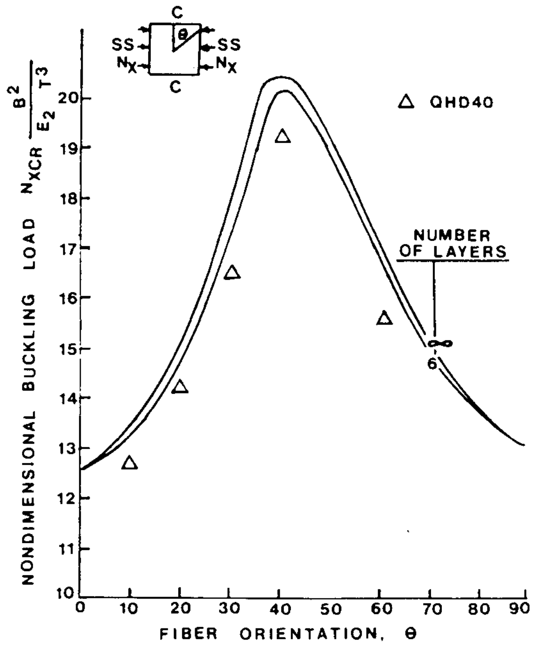


**Fig. 2 Display of critical buckling loads for antisymmetric angle-ply square plate vs ply orientation with simply supported/clamped boundary conditions (material 2).**

ply composite plates. For these cases, the $E_1/E_2$ ratio is varied while the aspect ratio is held constant. The results are in good agreement to those obtained by Noor[7] as shown in Fig. 1.

Figure 2 displays buckling loads of antisymmetric angle-ply plates with simply supported/clamped boundary conditions for different ply orientation angles Θ. These are compared to those obtained by Sharma.[8] As can be observed, the present formulation yields an upper bound to the elasticity model. However, the overall results are good and the formulation is much less cumbersome than a three-dimensional approach.

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# Application of Diverging Motions to Calculate Loads for Oscillating Motions

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## Introduction

USUALLY, the unsteady aerodynamic forces needed to calculate flutter are obtained by using the calculation methods for purely harmonic motions. The increased use of active control technology has led to the development of calculation methods that also produce results for nonharmonic motion (exponentially diverging or converging motion).[1]

Meanwhile, the technique has become widely accepted to obtain similar results by the analytic continuation of a polynomial fit through aerodynamic loads for purely harmonic motion.[2] In this way, the accuracy of the approximation is strongly dependent of the accuracy of the aerodynamic loads for harmonic motion. However, for complex configurations, complex flow (transonic), and high frequencies, the accurate calculation of these loads is possible only at relatively high computer costs.

## Procedure

In order to reduce these costs, an alternative procedure is used which reflects a suggestion already made by Jones[3] in 1945. Consider an arbitrary motion with a time function $e^{st}$ in which the Laplace parameter $s=g+ik$ is complex in general. The procedure includes the following steps:

1) Obtain the aerodynamic data for a purely exponentially diverging motion (i.e., $g$ positive and $k=0$) instead of a harmonic motion as in the usual methods.

2) Make a polynomial fit through those data.

3) Suppose that the polynomial fit is valid throughout the complex $s$ plane.

The polynominal that is applied is widely used in active control studies is

$$L(s)=\sum_{n=0}^{2} a_n s^n + \sum_{n=3}^{m} a_n \Bigg/ \left(s+\frac{s_{\max}}{n-2}\right)$$

Fitting this polynomial is performed by means of a least squares procedure.

## Results

Results obtained with this new approach are presented in Figs. 1-3 for a rectangular wing ($AR=2$) that performs a diverging pitching motion about the midchord at a Mach number of 0.8. The wing lift coefficient $C_L$ and moment about the midchord coefficient $C_M$ (both real!) have been calculated with the computer code ARSPNSC developed at NLR. This code is an extension of the ARSPNS code described in Ref. 4 and is capable of treating arbitrary thick lifting and nonlifting bodies oscillating in subsonic flow. Figure 1 shows a comparison of some polynomial fits for a purely diverging ex-

Received Aug. 26, 1985; revision received Jan. 30, 1986. 

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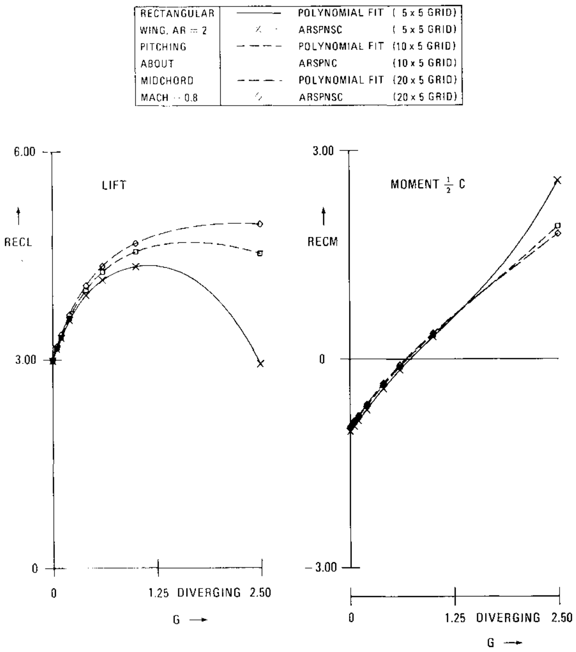


**Fig. 1 Comparison of ARSPNSC results and a polynominal fit for purely exponential diverging motion.**

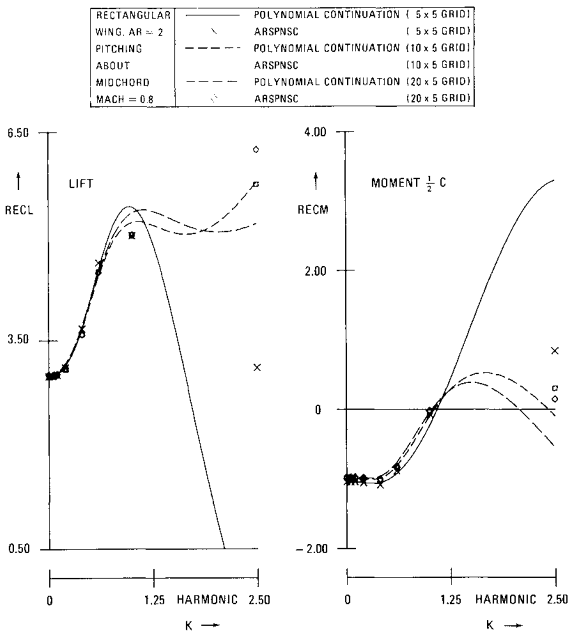


**Fig. 2 Demonstration of feasibility to obtain air loads due to harmonic motion from those due to diverging motion (real part).**

ponential motion (positive real value of $s$). The reference length is the semichord. The panel distributions used comprise 5, 10, or 20 panels of equal length along the chord and 5 panels of equal length along the semispan. Starting at $g = 0.5$, the influence of the panel size is clearly demonstrated. Figures 2 and 3 show a comparison of results for purely harmonic motion calculated directly with ARSPNSC and results obtained through $L(s)$ in which $m = 7$ and $s_{\max} = 5$ have been chosen. Now, of course, the lift and moment coefficients have real and imaginary parts.

Remarkably good agreement is obtained up to $k = 1.0$. Further reduction of the differences might also be obtained by using more points along the interval and by using polynomials that better simulate the behavior at larger frequencies (i.e., according to piston theory) or discrete numerical continuation techniques with a proper condition (radiation) at the border.

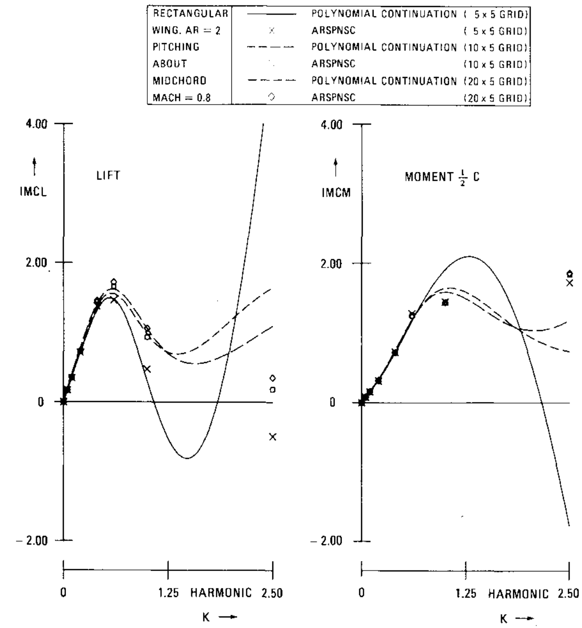


**Fig. 3 Demonstration of feasibility to obtain air loads due to harmonic motion from those due to diverging motion (imaginary part).**

The unfavorable influence of the panel size at increasing frequencies is probably due to the fact that, whatever the panel size, the numerical error will always dominate at large enough frequencies and may not be approximated appropriately by the polynomial given above. After all, the polynomial has never been designed to model numerical errors. Future work is needed to investigate the abovementioned possibilities further.

The proposed procedure yields probably the following advantages:

1) All calculations can be performed with real quantities, reducing the computational work by a factor as high as four by eliminating complex floating point operations in the determination of aerodynamic influence coefficients and the aerodynamic equations.

2) A factor of two reduction in memory requirements.

3) A reduction by a factor greater than one in the computational time by improving the convergence of the iteration procedure for solving the aerodynamic equations. More specifically, the convergence is improved by the diagonal dominancy of the equations, which increases with increasing real $s$ values. The reduction counts especially in case of transonic time-linearized methods.

4) A factor of four reduction in computational time involved in solving the aerodynamic equations with a direct method.

5) The generation of the polynomial fit through real data for a purely diverging motion seems to be much easier than through complex data corresponding to harmonic motions.

## Conclusion

An outline of a simple procedure has been presented to reduce the computational labor involved in determining unsteady aerodynamic forces by probably a factor of four. This procedure may help the aeroelastician to make cost-effective calculations of aerodynamic forces for complex flow (e.g., transonic) and for complex configurations (e.g., wing-bodies).

## Acknowledgment

The author thanks Mr. H. L. Hassig for bringing Ref. 3 to his attention during the preparation of this paper.

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# An Approach for Reducing Computational Requirements in Modal Identification


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## Sparse Time Domain Algorithm

LIKE other time domain modal identification techniques, the sparse time domain algorithm (STD) is based on using the free decay or impulse time response functions of the structure under test as the data for identification. Assuming linearity, or series expansion for a nonlinear system,[1] lumped parameter model representation of distributed parameter systems,[2] equivalent viscous damping and allowing for measurement noise, these responses are expressed as functions of the complex vectors $\psi$ and the characteristic roots $\lambda$ from which all modal parameters can be determined. If the initial condition's constants are implicitly contained in $\psi$, these responses are

$$\{x(t)\} = \sum_{i=1}^{2n} \{\psi_i\} e^{\lambda_i t} + \{n(t)\} \tag{1}$$

To reduce the effects of measurement noise on the identification accuracy, if these responses are sought to contain $n$ structural modes, an oversized identification model of $m$ degrees of freedom is used. In general, $m$ is greater than $n$ to an extent dependent on the noise-to-signal ratio of the measurements.

If the measurements are sampled at a sampling frequency of $f_s$ Hz and the time between samples is $\Delta t$ where

$$f_s = 1/\Delta t \tag{2}$$

a response matrix $[\phi]$, $(r \times 2m)$ where $r > 2m$, is constructed such that

$$\phi_{ij} = x_i\{k + (j-1)\ell\Delta t\} \tag{3}$$

$$i = 1,2,\ldots r \text{ and } j = 1,2,\ldots 2m$$

where $k$ is an arbitrary constant integer and $\ell$ a selected integer that is determined according to the antialiasing condition ($1/\ell\Delta t$ greater than twice the maximum frequency in the response). In matrix form, Eqs. (1) and (3) can be written as

$$[\phi] = [\psi]\ [\Lambda] \tag{4}$$

where

$$\Lambda_{ij} = e^{\lambda_i\{k + (j-1)\ell\Delta t\}}, \quad (i,j = 1,2,\ldots 2m)$$

In Eqs. (3) or (4), the measurement $x_i$ may be 1) the actual measurement on a test structure, 2) the pseudomeasurement that is an actual measurement delayed in time, 3) the actual or pseudomeasurement from a different test with different initial conditions (or initial excitation location), or 4) the actual or pseudomeasurements from a multiple excitation test.

To convert Eq. (4) to the desired eigenvalue problem, a similar response matrix $[\hat{\phi}]$ is constructed such that

$$\hat{\phi}_{ij} = \phi_{i,j+1} \tag{5}$$

and, by simple matrix algebra, it can be shown that

$$[\phi][\Lambda]^{-1}[\alpha] = [\hat{\phi}][\Lambda]^{-1} \tag{6}$$

$$[H][\Lambda]^{-1} = [\Lambda]^{-1}[\alpha] \tag{7}$$

where $[\alpha]$ is a diagonal matrix whose elements are

$$\alpha_i = e^{\lambda_i \ell\Delta t}$$

and $H$ satisfies the equation

$$[\phi][H] = [\hat{\phi}]$$

Because of the relation between $[\phi]$ and $[\hat{\phi}]$, the $[H]$ matrix takes the form

$$\begin{bmatrix} 0 & 0 & 0 & 0\ldots 0 & a_1 \\ 1 & 0 & 0 & 0\ldots 0 & a_2 \\ 0 & 1 & 0 & 0\ldots 0 & a_3 \\ 0 & 0 & 1 & 0\ldots 0 & a_4 \\ \hline \\ \hline 0 & 0 & 0 & 0\ldots 1 & a_{2m} \end{bmatrix} \tag{8}$$

which is a sparse upper Hessenberg matrix with only one column $\{a\}$ of information. To compute such a vector, the equation

$$[\phi]\{a\} = \{\hat{\phi}_{2m}\} \tag{9}$$

is used and, since Eq. (9) is an overdetermined system of equations, the least squares solution is used to give

$$[\phi]^T[\phi]\{a\} = [\phi]^T\{\hat{\phi}_{2m}\} \tag{10}$$

or

$$[B]\{a\} = \{b\} \tag{11}$$

and since $[B] = [\phi^T\phi]$, it is symmetrical and positive definite and Eq. (11) can be solved by using the stable Cholesky decomposition[3] to solve for vector $\{a\}$.

To reduce any bias errors associated with solving for $\{a\}$ that may arise because of the inherent statistically biased errors of the least squares solution, a double least squares solution may be performed. Such a step seems unnecessary since, as it will be shown later, the classical least squares ap-

Received March 8, 1985; presented as Paper 85-0811 at the AIAA/ASME/ASCE/AHS 26th Structures, Structural Dynamics and Materials Conference, April 15-17, 1985; revision received Nov. 11, 1985. 

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